

FERMION MASS PREDICTIONS IN A GENERALIZED EXTENDED TECHNICOLOUR SCENARIO

by

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Abstract

We review the recent discussion in the literature of one family extended technicolour models with techni-fermion mass spectra compatible with the experimental data for the precision parameters S, T, U, V, W and X and ETC interactions compatible with the LEP measurements of the $Z \rightarrow b\bar{b}$ vertex. To investigate whether these scenarios are consistent with the third family fermion masses we develop a generalized ETC model in which ETC interactions are represented by four Fermi interactions. We discuss in detail the reliability of the gap equation approximation to the non-perturbative dynamics. Two generic scenarios of couplings fit the precision data and third family masses; one is an unpreictive existence proof, the other, which generates the large top mass by direct top condensation, has a minimal number of interactions that break the global symmetry of the light fermions in the observed manner. This latter scenario makes surprisingly good predictions of the charm, strange and up quark masses.

1 Introduction

The Holy Grail of the next generation of accelerator experiments is a renormalizable, predictive model of the gauge boson and fermion masses that break electroweak symmetry. Models in which electroweak symmetry is broken by a condensate of strongly interacting fermions [1, 2] are an enticing possibility since they appeal to the successes of the BCS theory of superconductivity and chiral symmetry breaking in QCD. Whilst strongly interacting models such as technicolour [1] provide a simple explanation of electroweak symmetry breaking (EWSB) and the W and Z gauge boson masses, the diverse light fermion masses are much harder to understand. In the past theorists have tended to concentrate on building models that extended the basic technicolour scenario [3, 4, 5] to include the light fermion masses (extended technicolour models, ETC) as an existence proof that technicolour models can generate the diverse spectrum observed. Many of these models [4], by virtue of being existence proofs, have been very complicated having at least as many free parameters as there are elements in the light fermion mass matrices. The hope is that experimental discoveries will shed light on a simpler model along these lines which predicts some or all of the light fermion masses.

Recent precision tests of particle interactions below the Z mass from LEP experiments [8] and low energy atomic measurements [9] have tightly constrained the parameters in the low energy effective theory of the electroweak symmetry breaking sector. The effects of particles heavier than $M_Z/2$ (which are integrated from the effective theory probed at LEP) on low energy observables have been neatly summerized in terms of the parameters S,T,U,V,W and X [6, 7] as well as the deviation from the tree level prediction for the process $Z \rightarrow b\bar{b}$ [10, 11, 12]. This new data has been used to rule out many of the ETC models constructed prior to LEP. Recent work [12, 13, 14, 23] has concentrated on finding techni-fermion mass spectra and extended technicolour interactions that are consistent with the new precision data. The conclusion has been that ETC scenarios with light techni-fermions and ETC

interactions broken above 10 GeV still provide valid existence proofs of “realistic” strongly interacting models of EWSB.

In this paper we wish to investigate whether the recent precision data sheds light on the form of a simple predictive technicolour scenario. In Section 2 we review the analysis of the precision data and the conclusion as to the form a realistic techni-fermion mass spectrum must take. In Section 3 we introduce a generalized form of ETC model with a minimal number of new ETC interactions which we represent by four Fermi interactions. In order to simplify the initial treatment we set the weak mixing angles to zero and concentrate on the charged leptons and quarks. The CKM matrix elements and neutrino mass generation are extremely model dependent; we stress that we make this approximation in order to generate generic statements about ETC. We discuss how in principle such a model could be predictive. We also introduce a model of the existence proof type in order to show that ETC can be coerced to fit any fermion mass data given sufficient new parameters (this scenario is completely unproductive but included for completeness). To perform a numerical search of the parameter spaces of these models we must make some approximation to the full non-perturbative strong dynamics. We shall use the familiar gap equation approximation [15]. In Section 4 we review the successes and failures of the gap equation with some numerical examples. In Section 5 we present two general scenarios of ETC model with techni-fermion and third family mass spectra compatible with all available experimental data. One of these scenarios is entirely unproductive whilst the other, a simple ETC model with top condensation makes surprisingly good predictions for the up, charm and strange quark masses. We present these predictions in Section 6. Finally in Section 7 we conclude by discussing the implications of our generalized model for ETC model building and the need to extend the analysis to the neutrino sector and the CKM matrix elements.

2 Precision Constraints On ETC

Recent precision LEP data [8] and low energy atomic physics measurements provide stringent constraints on the physics responsible for EWSB. In this section we review these constraints and the results of Refs [12, 13, 14, 23] which suggest ETC models compatible with these constraints may exist. We divide our discussion of these constraints into two types: oblique corrections and non-oblique corrections. In addition we briefly discuss constraints from flavour changing neutral currents (FCNC) and a light pseudo-scalar spectrum.

2.1 Oblique Corrections

The major contributions to low energy observables from fermions and scalars with masses greater than $M_Z/2$ occur at one loop as oblique corrections to gauge boson propagators [17]. These corrections have been parameterized by Peskin and Takeuchi [6] and by Burgess *et al.* [7] in terms of the six parameters S,T,U,V,W and X. LEP's precision measurements have been performed on the Z mass resonance and hence the parameters associated with charged current interactions, U and W, are the least well constrained experimentally. A global fit [18] to the experimental data in which all six parameters S,T,U,V,W and X are allowed to vary simultaneously gives the one standard deviation bounds

$$\begin{aligned} S &\sim -0.93 \pm 1.7 & V &\sim 0.47 \pm 1.0 \\ T &\sim -0.67 \pm 0.92 & X &\sim 0.1 \pm 0.58 \end{aligned} \tag{2.1}$$

Explicit calculation [19] in ETC models gives the result $X \sim 0$ in all scenarios. The parameter V is only non-zero when a techni-fermion's mass is of order M_Z ($M \sim 50\text{GeV}$ $V \sim -0.15N_{TC}$; $M \sim 100\text{GeV}$ $V \sim -0.02N_{TC}$), where N_{TC} is the number of technicolours. If V and X both fall to zero the global fit to data for S and T is much more restrictive; the one standard deviation bounds are [18]

$$S \sim -0.5 \pm 0.6 \qquad T \sim -0.3 \pm 0.6 \qquad (2.2)$$

Calculating V,W and X for a strongly interacting doublet is difficult since these parameters measure the deviations from a Taylor expansion of the gauge boson self energies. Chiral models of strong interactions [20] in which the low energy effective theory is given as a derivative expansion are, therefore, completely inadequate. It is reasonable to assume that the strongly interacting results show the same behaviour as the weakly interacting results. In our model we shall assume that the techni-neutrino mass is $\sim 50 - 100 GeV$ so that V is non-zero and the less stringent bounds on S and T apply.

The contribution to the T parameter from a weakly interacting fermion doublet (U,D) with momentum independent mass is given approximately by the form [6]

$$T \simeq \frac{1}{12\pi s_{\theta_W}^2 c_{\theta_W}^2} \left[\frac{(\Delta m)^2}{M_Z^2} \right] \qquad (2.3)$$

where s_{θ_W} and c_{θ_W} are the sine and cosine of the weak mixing angle and Δm the mass splitting within the doublet. We conclude that techni-fermions with masses much greater than M_Z must be mass degenerate or else give too large a contribution to the T parameter. For example a doublet with mass splitting of $150 GeV$ contributes $T = 0.41 N_{TC}$, of $100 GeV$ contributes $T = 0.18 N_{TC}$, and of $50 GeV$ contributes $T = 0.04 N_{TC}$. The T parameter contribution from a strongly interacting doublet can be estimated in Dynamical Perturbation Theory [21] as

$$T = \frac{1.37}{F_{\pi^3}^2} (F_{\pi^\pm}^2 - F_{\pi^3}^2) \qquad (2.4)$$

where F_{π^\pm} and F_{π^3} are the charged and neutral techni-pion decay constants given by [22]

$$F_{\pi^3} = \frac{N_{TC}}{32\pi^2} \int_0^{\Lambda^2} dk^2 k^2 \left(\frac{\Sigma_U^2 - k^2(\Sigma_U^2)'/4}{(k^2 + \Sigma_U^2)^2} + U \leftrightarrow D \right) \quad (2.5)$$

$$F_{\pi^\pm} = \frac{N_{TC}}{32\pi^2} \int_0^{\Lambda^2} dk^2 k^2 \frac{F(k^2)}{(k^2 + \Sigma_U^2)(k^2 + \Sigma_D^2)} \quad (2.6)$$

$$\begin{aligned} F(k^2) = & (\Sigma_U^2 + \Sigma_D^2) - \frac{1}{4}k^2(\Sigma_U^2 + \Sigma_D^2)' - \frac{1}{8}[(\Sigma_U - \Sigma_D)^2]' \\ & - \frac{1}{4}(\Sigma_U - \Sigma_D)(\Sigma_U + \Sigma_D)'(\Sigma_U' \Sigma_D - \Sigma_U \Sigma_D') \\ & + [\frac{1}{2}k^2(\Sigma_U^2 - \Sigma_D^2) - \frac{1}{4}k^2(k^2 - \Sigma_U \Sigma_D)(\Sigma_U - \Sigma_D) \\ & \times (\Sigma_U + \Sigma_D)'] \left(\frac{1+(\Sigma_U^2)'}{(k^2 + \Sigma_U^2)} - \frac{1+(\Sigma_D^2)'}{(k^2 + \Sigma_D^2)} \right) \end{aligned} \quad (2.7)$$

where Σ_U and Σ_D are the self energies of the fermions and the prime indicates the derivative with respect to k^2 . These equations show the same behaviour as Eqn(2.3) with some enhancement [22] for a given mass splitting. We shall make use of them in our analysis of T below.

The contribution to the S parameter from a weakly interacting fermion doublet with momentum independent Dirac masses is given by the form [6]

$$S_{weak} = \frac{1}{6\pi} \left[1 - Y_L \ln \left(\frac{m_U^2}{m_D^2} \right) \right] \quad (2.8)$$

where Y_L is the left handed doublet's hypercharge. There has been much discussion in the literature [23] of how this result is affected by the inclusion of strong interactions for the doublet. In the Non Local Chiral Model (NLCM) of strong interactions in Ref[24] the S parameter may be expressed as an integral equation over the techni-fermions' self energies. As these self energies deviate from being momentum independent (as is suggested by gap equation solutions [15]) the contribution to S rises in the custodial SU(2) limit. Walking technicolour theories [16] and models with strong ETC interactions (such as we shall have below) which enhance the high momentum tail of the self energies [15] will presumably give

contributions to S that lie between the highest estimate of the NLCM and the perturbative result. It is also unclear whether custodial $SU(2)$ violation in the high momentum tails of the doublet's self energy is sufficient to give negative contributions to S . We shall adopt as an upper bound on the contribution for a techni-doublet

$$S_{strong} = N_{TC}[S_{weak} + 0.05] \quad (2.9)$$

where N_{TC} is the number of technicolours and where we calculate S_{weak} using the techni-fermions mass (given by $\Sigma(m) = m$). This result agrees with the observed data for QCD (the custodial $SU(2)$ limit) and with an analysis of the contribution to S from a techni-lepton doublet with a small Majorana mass perturbing the custodial isospin limit [14]. In addition we note that this result is the most conservative estimate of S in the literature away from the custodial isospin limit (it reduces the negative contributions from doublets with mass splittings). In a one family technicolour model such as we shall be considering below in which the techni-fermions are all mass degenerate we obtain $S = 0.4N_{TC}$ which is in excess of the experimental limit for all but the most minimal technicolour groups. To reduce this value we require doublets with mass splittings, however, we must be careful not to violate the T parameter bound.

These results lead to a one family technicolour techni-fermion mass spectra of the form [13, 14]

$$m_Q \sim \text{degenerate}, \quad m_E \sim 150 - 250 \text{ GeV}, \quad m_N \sim 50 - 100 \text{ GeV} \quad (2.10)$$

Perturbatively these doublets would give $S \sim 0.09N_{TC}$, $T \sim 0.3N_{TC}$ and $V \sim -(0.15 - 0.02)N_{TC}$. Our non-perturbative upper bound on S is thus $0.29N_{TC}$. We conclude that this techni-fermion spectrum probably lies within the experimental constraints for $N_{TC} < 6$. In addition we note that a large Majorana neutrino mass for the techni-neutrino gives a

somewhat larger negative contribution to S and a smaller contribution to T [14].

2.2 Non-Oblique Corrections

The ETC gauge bosons responsible for the light fermion masses give rise to non-oblique corrections to fermion anti-fermion production rates at LEP [10]. If the ETC interactions are orthogonal to the standard model gauge group then these non-oblique effects serve to correct the left handed fermion couplings by

$$\delta g_L^{ETC} \sim -\frac{1}{2} \frac{g_{ETC}^2}{M_{ETC}^2} F_\pi^2 \frac{e}{s_{\theta_W} c_{\theta_W}} I_3 \quad (2.11)$$

where g_{ETC} and M_{ETC} are the ETC gauge boson coupling and mass respectively, I_3 is the external fermion's weak isospin and F_π is the electroweak symmetry breaking scale. Only the coupling of the ETC gauge boson, g_{ETC}^2/M_{ETC}^2 , that is responsible for the top quark's mass is sufficiently large for the experimental data to constrain. These non-oblique effects are potentially visible in the $Z \rightarrow b\bar{b}$ vertex, measured by the ratio of Z boson decay widths to $b\bar{b}$ over that to all non- $b\bar{b}$ hadronic final states [10]

$$\Delta_R = \frac{\delta(\Gamma_b/\Gamma_{h \neq b})}{\Gamma_b/\Gamma_{h \neq b}} \sim \frac{2\delta g_L g_L}{g_L^2 + g_R^2} \quad (2.12)$$

where $g_L = \frac{e}{s_\theta c_\theta}(-\frac{1}{2} + \frac{1}{3}s_\theta^2)$, $g_R = \frac{e}{s_\theta c_\theta}(\frac{1}{3}s_\theta^2)$.

If the top quark mass ($m_t > 130\text{GeV}$) is generated by a perturbative ETC gauge boson (ie $g_{ETC}^2 \sim 1$) then the ETC breaking scale must be of order 1TeV. The ETC contribution to $\Delta_R \sim 4\%$ [10, 11] is approximately double the maximum experimentally consistent value [8]. However, if the ETC coupling is allowed to rise to 40 – 80% of it's critical coupling ($g_C^2 = 8\pi^2$) at a breaking scale of 10TeV then a physical top mass can be obtained for a realistic value of Δ_R [12]. We shall, therefore, take the lightest ETC gauge boson to have mass $M_{ETC} \sim 10\text{TeV}$.

2.3 Other Experimental Constraints

There are two additional constraints on ETC models, flavour changing neutral currents (FCNC), and the large, potentially light, pseudo Goldstone boson spectrum associated with the $SU(8)_L \otimes SU(8)_R \rightarrow SU(8)_V$ global chiral symmetry breaking of the techni-fermions. We shall briefly review these problems in this section

FCNCs [1, 3] arise in ETC models through the interactions of the massive gauge bosons associated with the breaking $SU(N+3)_{ETC} \rightarrow SU(N)_{TC}$ + three light families. Each of the light fermions has an associated ETC coupling, g_{ETC}^2/M_{ETC}^2 , given by

$$g_{ETC}^2/M_{ETC}^2 \sim m_f/\Lambda_{TC}^3 \quad (2.13)$$

where m_f is the fermion's mass. An analysis of the contributions to FCNCs in Ref[25] assuming that any FCNC involving a particular light fermion have a coupling at least as small as the calculated value in Eqn(2.13) reveals no constraints on the model from FCNCs. In addition we note that in models such as those we discuss below with strong ETC interactions the ETC coupling in Eqn(2.13) can be a considerable over estimate and hence FCNCs will be suppressed further. Thus although the contributions to FCNCs are model dependent models [4] do exist in the literature which naturally avoid FCNC constraints.

ETC models with a full techni-family give rise to 60 light pseudo Goldstone bosons (PGB) and 3 massless Goldstone bosons associated with the 63 broken generators of the techni-fermions' approximate global chiral symmetry [26]. The 60 PGBs acquire masses through the standard model and ETC interactions that perturb the global symmetry group. Calculation [26] of the PGB's masses from the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ interactions of the techni-fermions suggests that as many as 7 PGBs may have masses below the current experimental search limits. However, the major source of global symmetry breaking in the standard model comes from the fermion masses generated in ETC models by the ETC

interactions. Calculation [27] of the contribution to the PGB masses from ETC interactions sufficiently strong to generate the observed light fermion mass spectra reveal that all the PGBs will have masses in excess of the current direct search limits. The only exception is the neutral PGB with constituent techni-neutrinos. However, neutrino mass generation is extremely model dependent and in the absence of a convincing model of the neutrino sector we argue that it is not possible to place an upper constraint on the PGB mass. These calculations suggest that ETC models are unconstrained by the PGB spectrum.

In addition to the usual PGB spectrum the authors of Ref[28] have argued that when ETC interactions grow close to their critical values there will be additional light (relative to M_{ETC}), scalar, ETC bound states of the light fermions. These bound states' masses will fall to $\sim 2m_f$, where m_f is the mass of the constituent fermion, as the ETC interactions grow to their critical values. The strongest ETC interactions in our models below ($\sim 80 - 90\%$ of g_C), which will presumably give rise to the lightest scalar spectrum, are associated with the top mass generation. We, therefore, expect the lightest such scalar to have a mass $> 100\text{GeV}$.

3 A Generalized ETC Model

We wish to study the viability of a range of ETC models without restricting to any particular scenario. Since we have argued that the experimental constraints restrict models to an ETC breaking scale of 10TeV or greater it will be a good approximation to model the ETC interactions by simple four Fermi operators (we expect higher dimensional operators to be sufficiently suppressed). Thus our general model will consist of an $SU(N)$ technicolour group and, in principle, any number of gauge invariant four Fermi operators acting on a full techni-family (N, E, U^c, D^c : c is a colour index). In addition we consider the third family of fermions which are technicolour singlets but interact with the techni-fermions by ETC interactions again modelled by four Fermi operators. The technicolour group becomes strongly interacting at the scale $\Lambda_{TC} \sim 1\text{TeV}$ forming techni-fermion condensates and breaking electroweak

symmetry. We shall allow the ETC charges to vary over all possible values and search for a general model(s) compatible with the experimental data discussed in Section 3 and the third family fermion masses. These solutions will hopefully provide a general basis from which to build more specific (renormalizable) models.

The ETC interactions in our model can be split in to two catagories, sideways and horizontal. Sideways interactions feed the techni-fermion condensates down to the light three families of fermions. There are four such operators connecting the techni-fermions and third family

$$\frac{g_{\nu_3}^2}{M_{ETC}^2} \bar{\Psi}_L N_R \bar{\nu}_\tau \psi_L \quad \frac{g_\tau^2}{M_{ETC}^2} \bar{\Psi}_L E_R \bar{\tau} \psi_L \quad \frac{g_t^2}{M_{ETC}^2} \bar{Q}_L U_R \bar{t} q_L \quad \frac{g_b^2}{M_{ETC}^2} \bar{Q}_L D_R \bar{b} q_L \quad (3.1)$$

where $\Psi = (N, E)$, $\psi = (\nu_\tau, \tau)$, $Q = (U, D)$ and $q = (t, b)$. For readers who wish to have a renormalizable ETC model in mind these correspond to operators generated by breaking $SU(N+1)_{ETC} \rightarrow SU(N)_{TC} + \text{third family}$ at the scale $M_{ETC} \sim 10 TeV$.

Horizontal interactions correspond to techni-fermion and light fermion self interactions of the form

$$\frac{g_f^2}{M_{ETC}^2} \bar{F}_L f_R \bar{f}_R F_L \quad (3.2)$$

where F is the left handed doublet containing the general fermion f and where there may in general be such an interaction for each fermion in the model. We might expect the third family fermions and their respective techni-fermion counter parts to share quantum numbers and hence horizontal interactions. Our models will respect this constraint except when direct top condensation is investigated. Again the reader may envision that these interactions are generated at the scale M_{ETC} perhaps most naively by the breaking of an additional $U(1)$ gauge group (allowing for the different fermions within a family to have different horizontal charges). We also note that all the four Fermi operators will have charges below their critical

couplings hence we may skip any discussion of the strong properties of isolated U(1) gauge interactions.

A realistic ETC scenario must agree with experimental data for m_{ν_τ} , m_τ , m_t , m_b , V,T and S. The general model has 8 independent four Fermi charges (12 if we allow the third family horizontal interactions to differ from their techi counter parts) and, therefore, we might expect solutions to exist compatible with the data. In Section 5 we will verify that such a solution exists, however, it is clearly unproductive. It is interesting to propose the minimal model in principle capable of reproducing the fermion mass spectra and test it for mass predictions.

To simplify the initial analysis of this paper we shall neglect the discussion of the neutrino masses in the model since their masses do not fit any obvious pattern in relation to the other light fermion masses. We effectively assume that there are no right handed neutrinos though we maintain right handed techni-neutrinos. The precise mechanism for suppressing neutrino masses is extremely model dependent. In addition since there is no reliable method of estimating the contribution to the S parameter from strongly interacting doublets we shall simply set the techni-neutrino mass to 50-100GeV in the future discussion and assume that a realistic S parameter is obtained provided the techni-electron mass lies between 150-250GeV as discussed in Section 2. The W and Z gauge boson masses will be dominated by the heavier techni-quarks and hence neglecting the details of the neutrino sector will have little effect on the technicolour dynamics. The T parameter, however, will presumably be dominated by the techni-lepton sector as in the techni-fermion spectra discussed in Section 2. We assume that the techni-lepton sector contributes $T \leq 1$ and hence the T parameter contribution from the techni-quarks must at most be a few tenths.

In addition we note that the CKM matrix elements only significantly vary from the identity for the first (lightest) family of fermions whose masses are generated by the weakest ETC operators. We conclude that quark mixings and CP violation are generated by those

weak interactions and, therefore, in discussion of the heavier two generations of fermions we may neglect the CKM matrix elements. There is no clear understanding of the origin of the CKM matrix elements and hence we wish to neglect their generation in this discussion since we wish to make model independent predictions. Making this approximation will clearly upset any predictions of the first family masses which are associated with large mixings and indeed in Section 6 we shall see this manifest.

Now we may consider the minimal number of ETC interactions necessary to generate the light fermion masses [29]

$$\begin{aligned}
m_t &= 160 \pm 30 GeV & m_b &= 5.0 \pm 0.3 GeV & m_\tau &= 1.784 GeV \\
m_c &= 1.5 \pm 0.2 GeV & m_s &= 0.2 \pm 0.1 GeV & m_\mu &= 0.105 GeV \\
m_u &= 5 \pm 3 MeV & m_d &= 10 \pm 5 MeV & m_e &= 0.51 MeV
\end{aligned} \tag{3.3}$$

The EWSB scale is set by the technicolour dynamics corresponding to the scale Λ_{TC} at which the technicolour group becomes strongly interacting. The third family masses are suppressed relative to this scale by a factor of ~ 10 , the second family by a further factor of $\sim 10 - 100$ and the first family by yet a further factor of $\sim 10 - 100$. It is natural to associate each generation with a separate sideways interaction (introducing a single additional interaction parameter for each family). The quarks in each family are more massive than the leptons so we must break the symmetry between them by the addition of at least one extra interaction; we shall introduce a single horizontal interaction for the quarks. Finally we notice that in the heaviest two families the top type quarks are more massive than the bottom type (for the moment we ignore the up down mass inversion since it is associated with the scale at which the approximation that the CKM matrix is the identity breaks down) and hence there must be an additional interaction on these quarks to break the symmetry between them; we introduce a single additional horizontal interaction for top type quarks.

There must be a minimum of 5 new interactions in our model to break the global symme-

tries that would otherwise leave the light fermions degenerate. Indeed it is hard to imagine how any model of the light fermion masses could have fewer free parameters than this.

4 The Devil We Know - The Gap Equation

Before we can discuss the success or failure of scenarios such as those discussed in Section 3 we must have a reliable method of calculating physical quantities in strongly interacting theories. The infinite tower of Schwinger Dyson equations are untractable so it is traditional to truncate the tower after the fermion two point function and replace other propagators and vertices with the perturbative Feynman rule. We then obtain the two gap equations [15] for the fermion self energy from SU(N) gauge interactions (in Landau gauge and with a running gauge coupling) and four Fermi interactions respectively

$$\Sigma(p) = \frac{3C(R)}{4\pi} \int_0^{\Lambda^2} \alpha(Max(k^2, p^2)) \frac{k^2 dk^2}{Max(k^2, p^2)} \frac{\Sigma(k)}{k^2 + \Sigma^2(k)} \quad (4.1)$$

$$\Sigma(p) = \frac{g^2}{8\pi^2 \Lambda^2} \int_0^{\Lambda^2} k^2 dk^2 \frac{\Sigma(k)}{k^2 + \Sigma^2(k)} \quad (4.2)$$

where C(R) is the casimir operator of the fermion's representation of the gauge group, α the running gauge coupling, g the four Fermi interaction strength, and Λ the UV cut off.

The major success of these gap equations is that they show chiral symmetry breaking behaviour [15]. In each case there is some critical coupling below which the solution to the equation is $\Sigma(k^2) = 0$ and above which $\Sigma(k^2) \neq 0$. Clearly, however, in the case of the gauge coupling the precise value of the critical coupling and the form of the solution depend upon the form of the running of the coupling both in the high momentum regime (where the

running may be calculated in perturbation theory and is known to depend on the number of interacting fermions) and in the non-perturbative regime.

In order to investigate the consistency of solutions within the gap equation approximation let us consider the minimal predictive ETC model proposed in Section 3. The gap equations for the techni-family and third family are

(4.3)

where $D(R)$ is the dimension of the techni-fermions' representation under the technicolour group.

The scale Λ_{TC} is determined by requiring the correct Z mass which is given by the techni-pion decay constant, F_π , in Eqn(2.5-7). For simplicity we neglect the mass splitting within the lepton doublet in the calculation of F_π ; since the Z mass is dominated by the techni-quark contribution to F_π this will introduce only small errors and allows us to avoid the complication of specifying the neutrino sector. The three four Fermi couplings are determined, for a given value of M_{ETC} , by requiring that the correct tau, top and bottom masses are obtained as solutions. We tune to two significant figures in the fermion masses and use $m_t \sim 170 GeV$ as a representative value. We cut the integrals off at M_{ETC} .

The dependence of the solutions on the N_{TC} , M_{ETC} and the running of the coupling both in the perturbative and non-perturbative regimes may now be investigated. We begin by allowing the technicolour coupling to run according to the one loop β -function result above Λ_{TC} and cut off the running below Λ_{TC}

$$\begin{aligned} \alpha(q^2) &= 2\alpha_C & q^2 < \Lambda_{TC} \\ \alpha(q^2) &= \frac{2\alpha_C}{1+2\alpha_C\beta\ln(q/\Lambda_{TC})} & q^2 > \Lambda_{TC} \end{aligned} \tag{4.4}$$

where α_C is the critical coupling in the fixed point theory ($\alpha_C = \pi/3C(R)$). We set $\beta = 1$, a typical running value and $M_{ETC} = 10TeV$. We assume that the techni-fermions lie in the fundamental representation of the technicolour group. In Fig 1 we show results for the techni-fermion self energies as a function of momenta for $N_{TC} = 3$ and 6. In Fig 2 we display the dependence of the techni-up quark's self energy in the $SU(3)_{TC}$ scenario to changes in the β -function for $M_{ETC} = 10TeV$. If the β -function falls below 0.22 then Λ_{TC} must be reduced below 100GeV which is presumably unphysical. In Fig 3 we show the low energy structure of the techni-up quark's self energy for varying ETC scales ($M_{ETC} = 5, 10$ and $50TeV$) again with $N_{TC} = 3$ and $\beta = 1$. The couplings that satisfy all these solutions are given in Table 1.

Fig 1: The solutions to the gap equations of Eqn(4.3) for the techni-fermion self energies with $N_{TC} = 3$ (solid curves) and $N_{TC} = 6$ (dashed curves). $M_{ETC} = 10TeV$ and $\beta = 1$. In each case the highest curve is the techni-up self energy, the middle curve the techni-down self energy and the lower curve the techni-electron self energy. The solutions are given by tuning the couplings to M_Z, m_t, m_b and m_τ .

Fig 2: Dependence of gap equation solutions in Eqn(4.3) for the techni-up self energy on the technicolour β -function with $N_{TC} = 3$ and $M_{ETC} = 10TeV$.

Fig 3: Dependence of gap equation solutions for the techni-up self energy in Eqn(4.3) on

$$M_{ETC} \text{ with } N_{TC} = 3 \text{ and } \beta = 1.$$

The ansatz for the running of α in the non-perturbative regime in Eqn(4.4) is only determined in as much as it must be finite at $q = 0$. In Fig 4 we compare the effects of two extreme choices for this regime. The first ansatz assumes that the coupling flattens out quickly at low momenta having the form of Eqn(4.4) but taking a maximum value of $1.5\alpha_C$. This ansatz is probably an underestimate of the coupling strength since there is a large discrepancy between Λ_{TC} and $\Sigma(0)$. The second ansatz assumes that outside the perturbative regime the coupling rises sharply from α_C to a maximum value of $3\alpha_C$

$$\begin{aligned} \alpha(q^2) &= 3\alpha_C & q^2 < \Lambda_{TC} \\ \alpha(q^2) &= \frac{\alpha_C}{1+\alpha_C\beta\ln(q/\Lambda_{TC})} & q^2 > \Lambda_{TC} \end{aligned} \tag{4.5}$$

this presumably is a somewhat over estimate of the coupling strength.

Fig 4: Dependence of gap equation solutions on the non perturbative running. Details of the coupling ansatzs are given in the text. $N_{TC} = 3$, $\beta = 1$ and $M_{ETC} = 10TeV$.

We observe that in each case the self energy solutions have the same general form though it is clearly impossible to distinguish the solutions phenomenologically. Although there is some variation in the shape of $\Sigma(k^2)$ the area under $\Sigma(k^2)$ that contributes to the light fermion masses are fixed (by the requirement that they give the correct Z mass) at least up to errors of at most order one. We therefore expect the light fermion masses we calculate in the gap equation approximation to a given ETC model to at least be representative of the rough pattern of masses the theory would produce. However, the precision electroweak parameters are plagued by error in this approximation. The T parameter is a measure of one percent differences between our calculated values of F_{π^3} and F_{π^\pm} which correspond to integrals over the self energies. Clearly this level of precision is not provided for. The calculated values for the techni-quark contribution to T in each of the above scenarios is given in Table 1 and vary between T=8.9 and T=24.2! Similarly we have argued that to achieve a realistic value of S and V we require the techni-electron mass (determined by the condition $\Sigma(M_E) = M_E$) lies in the range $150 - 250GeV$. The calculated value of M_E is given in Table 1 also and again we see a large variation, $M_E = 90 - 260GeV$. We shall only

be able to argue about the gross features of the techni-fermion spectra and on where these are compatible with the realistic mass spectra in Eqn(2.10).

	N_{TC}	α_{MAX}	β	M_{ETC}	Λ_{TC}	$g_3/g_C\%$	$g_Q/g_C\%$	$g_t/g_C\%$	T_Q	M_E/GeV
	3	2	1.00	10	0.60	40.0	51.8	52.66	16.4	170
FIG 1	6	2	1.00	10	0.20	13.4	33.2	35.1	20.4	90
FIG 2	3	2	0.75	10	0.50	41.6	49.5	50.1	17.5	160
	3	2	0.50	10	0.35	45.0	44.5	46.1	19.5	140
	3	2	0.22	10	0.10	52.9	29.3	36.0	24.2	90
FIG 3	3	2	1.00	5	0.52	30.4	50.64	60.8	19.0	150
	3	2	1.00	50	0.49	70.7	17.8	14.5	15.8	140
FIG 4	3	3	1.00	10	0.60	36.0	55.1	57.7	8.9	260
	3	1.5	1.00	10	1.10	48.4	38.3	47.8	22.9	100

Table 1: Numerical values of the couplings and scales used to plot Fig 1-4. The non-perturbative ansatz for the technicolour coupling is indicated by the maximum value α_{MAX} . The four Fermi couplings are given as percentages of the critical coupling ($g_C^2 = 8\pi^2$). T_Q is the contribution to T from the techni-quarks. Solutions are obtained by tuning parameters to give the correct Z mass, m_τ , m_b and m_t .

Finally we note that even if the gap equations are not a realistic approximation to the underlying Schwinger Dyson equations they still provide a parameterization of the techni-fermions' self energies. Thus whilst the gap equation couplings may not be physical the

existence of gap equation solutions consistent with the experimental data is indicative that couplings exist in the full theory also compatible with the data.

5 Successful Scenarios

Our analysis in Section 4 of the minimal predictive model proposed in Section 3 suggests that the techni-quarks in such a scenario give rise to too large a contribution to the T parameter ($T_Q \sim 15$, see Table 1). It is interesting to note however that the techni-fermion self energies (in Fig 1) show the general pattern of the realistic mass pattern in Eqn(2.10) except for the overly large splitting between the techni-up and techni-down quarks. In this section we present two scenarios in which the techni-up techni-down mass splitting lies within experimental constraints, one model is completely unproductive the other is a variation on the minimal predictive model with direct top condensation.

5.1 An Existence Proof

In principle the ETC couplings in the generalized ETC model described in Section 3 need not be related and we obtain the gap equations

(5.1)

The top and bottom quark masses within this general model are determined by their separate sideways interactions. Although the top and bottom masses feed back into the techni-fermions' self energies tending to enhance the techni-up self energy it is clear that the separate horizontal interactions on the top and bottom type quarks can be used to enhance the techni-bottom self energy to oppose this custodial SU(2) violating effect. We can tune a set of couplings to give $T_Q = 0$ and which correctly describe the Z, tau, top and bottom

masses eg a scenario with $g_E = g_U = 0$:

N_{TC}	α_{MAX}	β	M_{ETC}	Λ_{TC}	$g_\tau/g_C\%$	$g_b/g_C\%$	$g_t/g_C\%$	$g_D/g_C\%$	T_Q
3	2.0	1.00	10	0.5	48.4	5.9	70.1	85.5	0.0

which give the techni-fermion masses

$$M_U \sim 400GeV, \quad M_D \sim 400GeV, \quad M_E \sim 140GeV \quad (5.2)$$

Such a scenario is consistent with the techni-fermion mass spectrum in Eqn(2.10) and hence with all available experimental data. The renormalizable models of Ref[4] can give rise to precisely this spectrum of ETC interactions, however, the degeneracy of the techni-quarks (and hence the low T parameter) arises from a conspiracy in the four Fermi couplings which seems unnatural. Nevertheless this scenario does provide an existence proof for ETC models.

5.2 Direct Top Condensation

The minimal predictive model of Eqn(4.3) fails because the techni-up self energy must be enhanced by too much relative to the techni-down in order to generate the top bottom mass splitting. Recently there has been much discussion in the literature of direct top condensation [2] giving rise to the large top mass. Whilst top condensation on its own is plagued by difficulties of fine tuning in order not to generate too large a top mass (ruled out by the T parameter measurements) when the top is not the major source of EWS breaking such fine tuning problems need not exist. We can construct an ETC model with top condensation simply by removing the horizontal interaction on the techni-up quark in the minimal predictive model. Since the large top mass is no longer generated by the sideways ETC interactions there is less constraint upon the ETC breaking scale, M_{ETC} , from the

$Z \rightarrow b\bar{b}$ vertex measurements. We shall allow M_{ETC} to fall to 5 TeV. The gap equations are then

$$(5.3)$$

In Table 2 we show some solutions to these equations and their predictions for the contribution to the T parameter from the techni-quarks.

N_{TC}	α_{MAX}	β	M_{ETC}	Λ_{TC}	$g_3/g_C\%$	$g_Q/g_C\%$	$g_t/g_C\%$	T_Q	M_E/GeV
3	2	1.00	10	1.10	20.3	51.2	83.8	0.71	320
3	1.5	1.00	10	2.65	21.4	39.4	89.1	2.92	225
3	3	0.95	10	0.95	20.3	58.0	79.9	0.20	410
3	2	0.50	10	0.78	19.7	47.3	86.1	1.00	300
6	2	1.00	10	0.45	24.2	48.4	82.4	5.08	185
3	2	1.00	5	1.07	13.5	47.3	87.1	0.09	300
4	2	1.00	5	0.85	12.4	47.3	87.2	0.11	270
5	2	1.00	5	0.65	13.5	46.1	87.8	0.24	230

Table 2: Numerical values of the couplings and scales of solutions to Eqn(5.3). The non-perturbative ansatz for the technicolour coupling is indicated by the maximum value α_{MAX} . The four Fermi couplings are given as percentages of the critical coupling ($g_C^2 = 8\pi^2$). T_Q is the contribution to T from the techni-quarks. Solutions are obtained by tuning parameters to give the correct Z mass, m_τ , m_b and m_t .

The solutions with a low ETC scale seem consistent with the techni-fermion mass spectrum proposed in Section 2 though the techni-electron mass is somewhat high. Within the gap equation approximation it is certainly not possible to discount this scenario so we shall consider it a successful ETC model.

6 Quark Mass Postdictions

We have argued in Section 3 that a model of EWSB and the third family masses (excluding neutrinos) must have at least four couplings and hence can not be “postdictive” of the third family masses. However, it is conceivable that only one additional parameter need be added to generate the second family masses (a parameter that suppresses the second family masses relative to the third) since quark lepton and custodial isospin symmetry breaking already exist in the model. Similarly one additional parameter might suffice to suppress the first family masses below the second but of course our neglect of the CKM matrix elements which are substantial for the first family makes this seem less likely to be successful. In this section we investigate the possibility of such postdiction in the scenarios we have discussed above.

The “existence proof” scenario does not lend itself to postdiction since to follow the pattern of the model of the third family masses we could simply introduce additional sideways interactions for each new light fermion sufficient to generate their mass. There are no constraints on the couplings so they are unpredictable. The first and second family masses are at least two orders of magnitude smaller than the techni-fermion masses and hence any feedback of the light two families masses into the techni-fermion self energies are negligible and do not upset our calculations of S and T. Although unpredictable the scenario still provides an existence proof of a realistic ETC model.

The top condensation scenario however is potentially predictive as described above. We introduce the additional sideways interactions

which we would expect to be generated if there was a single breaking scale associated with each of the first and second families in the breaking of $SU(N+3)_{ETC} \rightarrow SU(N)_{TC}$ + three families. Again the feedback of the first and second family masses to the techni-fermions and third family are negligible. We set the coupling strength of the new sideways interaction by requiring that we generate the correct muon and electron masses. The up, down, charm and strange quark masses are now predictions of the model. Explicitly

$$\begin{aligned}
\Lambda_{TC} & \text{ determined by } M_Z & g_t & \text{ determined by } m_t \\
g_3 & \text{ determined by } m_\tau & g_2 & \text{ determined by } m_\mu \\
g_Q & \text{ determined by } m_b & g_1 & \text{ determined by } m_e
\end{aligned} \tag{6.1}$$

Although the predictions of the model are clear cut our ability to calculate is limited as discussed in Section 4. The gap equation solutions are, however, moderately well bounded since the integrals over the techni-fermion's self energies are fixed to a good degree by the imposed requirements that they correctly give the Z, tau, bottom and top masses. We shall quote the range of predictions from all the coupling values in Table 2 as an estimate of our theoretical errors. We obtain

$$\begin{aligned}
m_c &= 1.5 \pm 0.8 GeV, & m_s &= 0.32 \pm 0.02 GeV \\
m_u &= 6.6 \pm 3.7 MeV, & m_d &= 1.5 \pm 0.2 MeV
\end{aligned}
\tag{6.2}$$

We immediately notice that these predictions are in surprisingly good agreement with the observed mass spectra except for the down quark. The failure to predict the down quark mass however is to be expected since we have neglected the generation of the CKM matrix which has large elements for the first family. Conservatively we can conclude that ETC models with the minimal number of ETC interactions that are sufficient to break the global symmetry of the light fermions in the observed pattern seem capable of reproducing the pattern of the observed light fermion mass spectrum.

7 Conclusions

The precision data from LEP [8] has provided tight constraints on the form of models of EWSB. It has been argued [13, 14] that technicolour models with a single techni-family with a light techni-neutrino and degenerate techni-quarks give contributions to the S,T and V parameters that lie within the experimentally allowed bands. If the top mass is generated by strong ETC interactions broken above $10 TeV$ then the model will lie within the experimental limits on non-oblique corrections to the $Zb\bar{b}$ vertex as well [12]. As a first step towards a fully renormalizable, predictive model of EWSB we have investigated whether an ETC model can be compatible both with the precision data and the light fermion masses. To make this investigation we have used a generalized one family ETC model in which the ETC interactions are represented by four Fermi interactions.

To calculate within this generalized model we have used the gap equation approximation to the Schwinger Dyson equations. Unfortunately even within the gap equation approximation the solutions for the techni-fermions self energies, $\Sigma(k^2)$, are dependent on the precise form of the running of the technicolour coupling. The technicolour dynamics are fixed to some degree by the requirement that the model gives rise to the correct Z boson mass (given

by an integral equation over the self-energies). Calculation of the light fermion masses (also given by integral equations over the self-energies) are, therefore, moderately stable. However, the precision electroweak variables are very sensitive to shifts in for example $\Sigma(0)$ and are hence less well determined. Nevertheless we have argued that couplings exist in the generalized ETC model that very plausibly fit the experimental constraints.

Two scenarios in the generalized ETC model have been found consistent with the precision data and the third family fermion masses. The first is an unproductive existence proof in which sufficient ETC couplings are included that the fermion mass spectra may be tuned to match the data. The second scenario contains what we have argued is the minimum number of different strength ETC interactions required to break the global symmetry on the third family in the observed pattern. This model achieves a sufficiently large top mass by direct top condensation.

In order to obtain a large top mass in these models the ETC interactions must be tuned close to their critical values. The “fine tuning” is at worst of order 10%, corresponding in our results to our need to quote ETC couplings to three significant figures in order to tune to two significant figures in the light fermion masses. In fact the tuning is only this severe for the ETC couplings that generate the top mass. This degree of tuning may not be unnatural since gauge couplings naturally run between their critical value, g_C , and $\sim 0.1g_C$ over many orders of magnitude of momentum. Clearly any greater degree of fine tuning which, for example, would be associated with significantly increasing Λ_{ETC} , would be unsatisfactory.

The top condensing scenario may be minimally extended to the first and second families. The model then makes predictions for the up, down, charm and strange quark masses. Our calculation of these masses shows that the charm, strange and up quark mass predictions are consistent (up to errors due to uncertainty in the gap equation approximation) with the experimental values. The model does not reproduce the up down mass inversion observed in nature but we have argued that this is the result of our neglect of the mechanism for

the generation of the CKM matrix which has large elements for the first family quarks. In addition we have neglected a discussion of the neutrino sector since their masses do not fit any obvious pattern in the fermion mass spectra. In this paper we have concentrated on predictions which are potentially generic to ETC models. Clearly it would be of interest to continue the analysis to models of neutrino masses and the CKM matrix but such analysis would only serve to confuse the cleaner model of quarks and charged leptons.

Hopefully the successes of the generalized ETC model here will be translatable to a renormalizable ETC model. In this respect the proposal in Ref[15] that the quark lepton mass splittings may result from QCD interactions, corresponding to $g_Q^2 \rightarrow \alpha_{QCD}$ in the top condensate scenario, is appealing. At the EWSB scale $\alpha_{QCD}(M_Z^2)/\alpha_{QCD}^{crit} \sim 15\%$. Our analysis suggests (see Table 2) that g_Q/g_C needs to be of order 50% however. The value of g_Q/g_C can be reduced (see Table 1) by increasing the maximum value the technicolour coupling reaches in the non-perturbative regime, or by increasing N_{TC} or Λ_{ETC} or finally by decreasing the technicolour β -function towards a walking value. Unfortunately each of these changes tends to increase the T parameter contribution from the techni-quarks. The uncertainties in the gap equation analysis though does not preclude the possibility.

We conclude that our unpredicitive model provides an existence proof that ETC models exist which satisfy the stringent precision measurement bounds. The scenario with direct top condensation provides the tantalizing possibility that ETC models can be constructed that are predicitive.

Acknowledgements

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FERMION MASS POSTDICTIONS IN A GENERALIZED EXTENDED TECHNICOLOUR SCENARIO

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Abstract

We review the recent discussion in the literature of one family extended technicolour models with techni-fermion mass spectra compatible with the experimental data for the precision parameters S, T, U, V, W and X and ETC interactions compatible with the LEP measurements of the $Z \rightarrow b\bar{b}$ vertex. To investigate whether these scenarios are consistent with the third family fermion masses we develop a generalized ETC model in which ETC interactions are represented by four Fermi interactions. We discuss in detail the reliability of the gap equation approximation to the non-perturbative dynamics. Two generic scenarios of couplings fit the precision data and third family masses; one is an unpredicitive existence proof, the other, which generates the large top mass by direct top condensation, has a minimal number of interactions that break the global symmetry of the light fermions in the observed manner. This latter scenario makes surprisingly good predictions of the charm, strange and up quark masses.

1 Introduction

The Holy Grail of the next generation of accelerator experiments is a renormalizable, predictive model of the gauge boson and fermion masses that break electroweak symmetry. Models in which electroweak symmetry is broken by a condensate of strongly interacting fermions [1, 2] are an enticing possibility since they appeal to the successes of the BCS theory of superconductivity and chiral symmetry breaking in QCD. Whilst strongly interacting models such as technicolour [1] provide a simple explanation of electroweak symmetry breaking (EWSB) and the W and Z gauge boson masses, the diverse light fermion masses are much harder to understand. In the past theorists have tended to concentrate on building models that extended the basic technicolour scenario [3, 4, 5] to include the light fermion masses (extended technicolour models, ETC) as an existence proof that technicolour models can generate the diverse spectrum observed. Many of these models [4], by virtue of being existence proofs, have been very complicated having at least as many free parameters as there are elements in the light fermion mass matrices. The hope is that experimental discoveries will shed light on a simpler model along these lines which predicts some or all of the light fermion masses.

Recent precision tests of particle interactions below the Z mass from LEP experiments [8] and low energy atomic measurements [9] have tightly constrained the parameters in the low energy effective theory of the electroweak symmetry breaking sector. The effects of particles heavier than $M_Z/2$ (which are integrated from the effective theory probed at LEP) on low energy observables have been neatly summerized in terms of the parameters S,T,U,V,W and X [6, 7] as well as the deviation from the tree level prediction for the process $Z \rightarrow b\bar{b}$ [10, 11, 12]. This new data has been used to rule out many of the ETC models constructed prior to LEP. Recent work [12, 13, 14, 23] has concentrated on finding techni-fermion mass spectra and extended technicolour interactions that are consistent with the new precision data. The conclusion has been that ETC scenarios with light techni-fermions and ETC

interactions broken above 10 GeV still provide valid existence proofs of “realistic” strongly interacting models of EWSB.

In this paper we wish to investigate whether the recent precision data sheds light on the form of a simple predictive technicolour scenario. In Section 2 we review the analysis of the precision data and the conclusion as to the form a realistic techni-fermion mass spectrum must take. In Section 3 we introduce a generalized form of ETC model with a minimal number of new ETC interactions which we represent by four Fermi interactions. In order to simplify the initial treatment we set the weak mixing angles to zero and concentrate on the charged leptons and quarks. The CKM matrix elements and neutrino mass generation are extremely model dependent; we stress that we make this approximation in order to generate generic statements about ETC. We discuss how in principle such a model could be predictive. We also introduce a model of the existence proof type in order to show that ETC can be coerced to fit any fermion mass data given sufficient new parameters (this scenario is completely unproductive but included for completeness). To perform a numerical search of the parameter spaces of these models we must make some approximation to the full non-perturbative strong dynamics. We shall use the familiar gap equation approximation [15]. In Section 4 we review the successes and failures of the gap equation with some numerical examples. In Section 5 we present two general scenarios of ETC model with techni-fermion and third family mass spectra compatible with all available experimental data. One of these scenarios is entirely unproductive whilst the other, a simple ETC model with top condensation makes surprisingly good predictions for the up, charm and strange quark masses. We present these predictions in Section 6. Finally in Section 7 we conclude by discussing the implications of our generalized model for ETC model building and the need to extend the analysis to the neutrino sector and the CKM matrix elements.

2 Precision Constraints On ETC

Recent precision LEP data [8] and low energy atomic physics measurements provide stringent constraints on the physics responsible for EWSB. In this section we review these constraints and the results of Refs [12, 13, 14, 23] which suggest ETC models compatible with these constraints may exist. We divide our discussion of these constraints into two types: oblique corrections and non-oblique corrections. In addition we briefly discuss constraints from flavour changing neutral currents (FCNC) and a light pseudo-scalar spectrum.

2.1 Oblique Corrections

The major contributions to low energy observables from fermions and scalars with masses greater than $M_Z/2$ occur at one loop as oblique corrections to gauge boson propagators [17]. These corrections have been parameterized by Peskin and Takeuchi [6] and by Burgess *et al.* [7] in terms of the six parameters S,T,U,V,W and X. LEP's precision measurements have been performed on the Z mass resonance and hence the parameters associated with charged current interactions, U and W, are the least well constrained experimentally. A global fit [18] to the experimental data in which all six parameters S,T,U,V,W and X are allowed to vary simultaneously gives the one standard deviation bounds

$$\begin{aligned} S &\sim -0.93 \pm 1.7 & V &\sim 0.47 \pm 1.0 \\ T &\sim -0.67 \pm 0.92 & X &\sim 0.1 \pm 0.58 \end{aligned} \tag{2.1}$$

Explicit calculation [19] in ETC models gives the result $X \sim 0$ in all scenarios. The parameter V is only non-zero when a techni-fermion's mass is of order M_Z ($M \sim 50GeV$ $V \sim -0.15N_{TC}$; $M \sim 100GeV$ $V \sim -0.02N_{TC}$), where N_{TC} is the number of technicolours. If V and X both fall to zero the global fit to data for S and T is much more restrictive; the one standard deviation bounds are [18]

$$S \sim -0.5 \pm 0.6 \qquad T \sim -0.3 \pm 0.6 \qquad (2.2)$$

Calculating V,W and X for a strongly interacting doublet is difficult since these parameters measure the deviations from a Taylor expansion of the gauge boson self energies. Chiral models of strong interactions [20] in which the low energy effective theory is given as a derivative expansion are, therefore, completely inadequate. It is reasonable to assume that the strongly interacting results show the same behaviour as the weakly interacting results. In our model we shall assume that the techni-neutrino mass is $\sim 50 - 100 GeV$ so that V is non-zero and the less stringent bounds on S and T apply.

The contribution to the T parameter from a weakly interacting fermion doublet (U,D) with momentum independent mass is given approximately by the form [6]

$$T \simeq \frac{1}{12\pi s_{\theta_W}^2 c_{\theta_W}^2} \left[\frac{(\Delta m)^2}{M_Z^2} \right] \qquad (2.3)$$

where s_{θ_W} and c_{θ_W} are the sine and cosine of the weak mixing angle and Δm the mass splitting within the doublet. We conclude that techni-fermions with masses much greater than M_Z must be mass degenerate or else give too large a contribution to the T parameter. For example a doublet with mass splitting of $150 GeV$ contributes $T = 0.41 N_{TC}$, of $100 GeV$ contributes $T = 0.18 N_{TC}$, and of $50 GeV$ contributes $T = 0.04 N_{TC}$. The T parameter contribution from a strongly interacting doublet can be estimated in Dynamical Perturbation Theory [21] as

$$T = \frac{1.37}{F_{\pi^3}^2} (F_{\pi^\pm}^2 - F_{\pi^3}^2) \qquad (2.4)$$

where F_{π^\pm} and F_{π^3} are the charged and neutral techni-pion decay constants given by [22]

$$F_{\pi^3} = \frac{N_{TC}}{32\pi^2} \int_0^{\Lambda^2} dk^2 k^2 \left(\frac{\Sigma_U^2 - k^2(\Sigma_U^2)'/4}{(k^2 + \Sigma_U^2)^2} + U \leftrightarrow D \right) \quad (2.5)$$

$$F_{\pi^\pm} = \frac{N_{TC}}{32\pi^2} \int_0^{\Lambda^2} dk^2 k^2 \frac{F(k^2)}{(k^2 + \Sigma_U^2)(k^2 + \Sigma_D^2)} \quad (2.6)$$

$$\begin{aligned} F(k^2) = & (\Sigma_U^2 + \Sigma_D^2) - \frac{1}{4}k^2(\Sigma_U^2 + \Sigma_D^2)' - \frac{1}{8}[(\Sigma_U - \Sigma_D)^2]' \\ & - \frac{1}{4}(\Sigma_U - \Sigma_D)(\Sigma_U + \Sigma_D)'(\Sigma_U' \Sigma_D - \Sigma_U \Sigma_D') \\ & + [\frac{1}{2}k^2(\Sigma_U^2 - \Sigma_D^2) - \frac{1}{4}k^2(k^2 - \Sigma_U \Sigma_D)(\Sigma_U - \Sigma_D) \\ & \times (\Sigma_U + \Sigma_D)'] \left(\frac{1+(\Sigma_U^2)'}{(k^2 + \Sigma_U^2)} - \frac{1+(\Sigma_D^2)'}{(k^2 + \Sigma_D^2)} \right) \end{aligned} \quad (2.7)$$

where Σ_U and Σ_D are the self energies of the fermions and the prime indicates the derivative with respect to k^2 . These equations show the same behaviour as Eqn(2.3) with some enhancement [22] for a given mass splitting. We shall make use of them in our analysis of T below.

The contribution to the S parameter from a weakly interacting fermion doublet with momentum independent Dirac masses is given by the form [6]

$$S_{weak} = \frac{1}{6\pi} \left[1 - Y_L \ln \left(\frac{m_U^2}{m_D^2} \right) \right] \quad (2.8)$$

where Y_L is the left handed doublet's hypercharge. There has been much discussion in the literature [23] of how this result is affected by the inclusion of strong interactions for the doublet. In the Non Local Chiral Model (NLCM) of strong interactions in Ref[24] the S parameter may be expressed as an integral equation over the techni-fermions' self energies. As these self energies deviate from being momentum independent (as is suggested by gap equation solutions [15]) the contribution to S rises in the custodial SU(2) limit. Walking technicolour theories [16] and models with strong ETC interactions (such as we shall have below) which enhance the high momentum tail of the self energies [15] will presumably give

contributions to S that lie between the highest estimate of the NLCM and the perturbative result. It is also unclear whether custodial $SU(2)$ violation in the high momentum tails of the doublet's self energy is sufficient to give negative contributions to S . We shall adopt as an upper bound on the contribution for a techni-doublet

$$S_{strong} = N_{TC}[S_{weak} + 0.05] \quad (2.9)$$

where N_{TC} is the number of technicolours and where we calculate S_{weak} using the techni-fermions mass (given by $\Sigma(m) = m$). This result agrees with the observed data for QCD (the custodial $SU(2)$ limit) and with an analysis of the contribution to S from a techni-lepton doublet with a small Majorana mass perturbing the custodial isospin limit [14]. In addition we note that this result is the most conservative estimate of S in the literature away from the custodial isospin limit (it reduces the negative contributions from doublets with mass splittings). In a one family technicolour model such as we shall be considering below in which the techni-fermions are all mass degenerate we obtain $S = 0.4N_{TC}$ which is in excess of the experimental limit for all but the most minimal technicolour groups. To reduce this value we require doublets with mass splittings, however, we must be careful not to violate the T parameter bound.

These results lead to a one family technicolour techni-fermion mass spectra of the form [13, 14]

$$m_Q \sim \text{degenerate}, \quad m_E \sim 150 - 250 GeV, \quad m_N \sim 50 - 100 GeV \quad (2.10)$$

Perturbatively these doublets would give $S \sim 0.09N_{TC}$, $T \sim 0.3N_{TC}$ and $V \sim -(0.15 - 0.02)N_{TC}$. Our non-perturbative upper bound on S is thus $0.29N_{TC}$. We conclude that this techni-fermion spectrum probably lies within the experimental constraints for $N_{TC} < 6$. In addition we note that a large Majorana neutrino mass for the techni-neutrino gives a

somewhat larger negative contribution to S and a smaller contribution to T [14].

2.2 Non-Oblique Corrections

The ETC gauge bosons responsible for the light fermion masses give rise to non-oblique corrections to fermion anti-fermion production rates at LEP [10]. If the ETC interactions are orthogonal to the standard model gauge group then these non-oblique effects serve to correct the left handed fermion couplings by

$$\delta g_L^{ETC} \sim -\frac{1}{2} \frac{g_{ETC}^2}{M_{ETC}^2} F_\pi^2 \frac{e}{s_{\theta_W} c_{\theta_W}} I_3 \quad (2.11)$$

where g_{ETC} and M_{ETC} are the ETC gauge boson coupling and mass respectively, I_3 is the external fermion's weak isospin and F_π is the electroweak symmetry breaking scale. Only the coupling of the ETC gauge boson, g_{ETC}^2/M_{ETC}^2 , that is responsible for the top quark's mass is sufficiently large for the experimental data to constrain. These non-oblique effects are potentially visible in the $Z \rightarrow b\bar{b}$ vertex, measured by the ratio of Z boson decay widths to $b\bar{b}$ over that to all non- $b\bar{b}$ hadronic final states [10]

$$\Delta_R = \frac{\delta(\Gamma_b/\Gamma_{h \neq b})}{\Gamma_b/\Gamma_{h \neq b}} \sim \frac{2\delta g_L g_L}{g_L^2 + g_R^2} \quad (2.12)$$

where $g_L = \frac{e}{s_\theta c_\theta}(-\frac{1}{2} + \frac{1}{3}s_\theta^2)$, $g_R = \frac{e}{s_\theta c_\theta}(\frac{1}{3}s_\theta^2)$.

If the top quark mass ($m_t > 130 GeV$) is generated by a perturbative ETC gauge boson (ie $g_{ETC}^2 \sim 1$) then the ETC breaking scale must be of order 1TeV. The ETC contribution to $\Delta_R \sim 4\%$ [10, 11] is approximately double the maximum experimentally consistent value [8]. However, if the ETC coupling is allowed to rise to 40 – 80% of it's critical coupling ($g_C^2 = 8\pi^2$) at a breaking scale of 10TeV then a physical top mass can be obtained for a realistic value of Δ_R [12]. We shall, therefore, take the lightest ETC gauge boson to have mass $M_{ETC} \sim 10TeV$.

2.3 Other Experimental Constraints

There are two additional constraints on ETC models, flavour changing neutral currents (FCNC), and the large, potentially light, pseudo Goldstone boson spectrum associated with the $SU(8)_L \otimes SU(8)_R \rightarrow SU(8)_V$ global chiral symmetry breaking of the techni-fermions. We shall briefly review these problems in this section

FCNCs [1, 3] arise in ETC models through the interactions of the massive gauge bosons associated with the breaking $SU(N+3)_{ETC} \rightarrow SU(N)_{TC}$ + three light families. Each of the light fermions has an associated ETC coupling, g_{ETC}^2/M_{ETC}^2 , given by

$$g_{ETC}^2/M_{ETC}^2 \sim m_f/\Lambda_{TC}^3 \quad (2.13)$$

where m_f is the fermion's mass. An analysis of the contributions to FCNCs in Ref[25] assuming that any FCNC involving a particular light fermion have a coupling at least as small as the calculated value in Eqn(2.13) reveals no constraints on the model from FCNCs. In addition we note that in models such as those we discuss below with strong ETC interactions the ETC coupling in Eqn(2.13) can be a considerable over estimate and hence FCNCs will be suppressed further. Thus although the contributions to FCNCs are model dependent models [4] do exist in the literature which naturally avoid FCNC constraints.

ETC models with a full techni-family give rise to 60 light pseudo Goldstone bosons (PGB) and 3 massless Goldstone bosons associated with the 63 broken generators of the techni-fermions' approximate global chiral symmetry [26]. The 60 PGBs acquire masses through the standard model and ETC interactions that perturb the global symmetry group. Calculation [26] of the PGB's masses from the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ interactions of the techni-fermions suggests that as many as 7 PGBs may have masses below the current experimental search limits. However, the major source of global symmetry breaking in the standard model comes from the fermion masses generated in ETC models by the ETC

interactions. Calculation [27] of the contribution to the PGB masses from ETC interactions sufficiently strong to generate the observed light fermion mass spectra reveal that all the PGBs will have masses in excess of the current direct search limits. The only exception is the neutral PGB with constituent techni-neutrinos. However, neutrino mass generation is extremely model dependent and in the absence of a convincing model of the neutrino sector we argue that it is not possible to place an upper constraint on the PGB mass. These calculations suggest that ETC models are unconstrained by the PGB spectrum.

In addition to the usual PGB spectrum the authors of Ref[28] have argued that when ETC interactions grow close to their critical values there will be additional light (relative to M_{ETC}), scalar, ETC bound states of the light fermions. These bound states' masses will fall to $\sim 2m_f$, where m_f is the mass of the constituent fermion, as the ETC interactions grow to their critical values. The strongest ETC interactions in our models below ($\sim 80-90\%$ of g_C), which will presumably give rise to the lightest scalar spectrum, are associated with the top mass generation. We, therefore, expect the lightest such scalar to have a mass $> 100GeV$.

3 A Generalized ETC Model

We wish to study the viability of a range of ETC models without restricting to any particular scenario. Since we have argued that the experimental constraints restrict models to an ETC breaking scale of 10TeV or greater it will be a good approximation to model the ETC interactions by simple four Fermi operators (we expect higher dimensional operators to be sufficiently suppressed). Thus our general model will consist of an $SU(N)$ technicolour group and, in principle, any number of gauge invariant four Fermi operators acting on a full techni-family (N, E, U^c, D^c : c is a colour index). In addition we consider the third family of fermions which are technicolour singlets but interact with the techni-fermions by ETC interactions again modelled by four Fermi operators. The technicolour group becomes strongly interacting at the scale $\Lambda_{TC} \sim 1TeV$ forming techni-fermion condensates and breaking electroweak

symmetry. We shall allow the ETC charges to vary over all possible values and search for a general model(s) compatible with the experimental data discussed in Section 3 and the third family fermion masses. These solutions will hopefully provide a general basis from which to build more specific (renormalizable) models.

The ETC interactions in our model can be split in to two catagories, sideways and horizontal. Sideways interactions feed the techni-fermion condensates down to the light three families of fermions. There are four such operators connecting the techni-fermions and third family

$$\frac{g_{\nu_3}^2}{M_{ETC}^2} \bar{\Psi}_L N_R \bar{\nu}_\tau \psi_L \quad \frac{g_\tau^2}{M_{ETC}^2} \bar{\Psi}_L E_R \bar{\tau} \psi_L \quad \frac{g_t^2}{M_{ETC}^2} \bar{Q}_L U_R \bar{t} q_L \quad \frac{g_b^2}{M_{ETC}^2} \bar{Q}_L D_R \bar{b} q_L \quad (3.1)$$

where $\Psi = (N, E)$, $\psi = (\nu_\tau, \tau)$, $Q = (U, D)$ and $q = (t, b)$. For readers who wish to have a renormalizable ETC model in mind these correspond to operators generated by breaking $SU(N+1)_{ETC} \rightarrow SU(N)_{TC} + \text{third family}$ at the scale $M_{ETC} \sim 10T\epsilon V$.

Horizontal interactions correspond to techni-fermion and light fermion self interactions of the form

$$\frac{g_f^2}{M_{ETC}^2} \bar{F}_L f_R \bar{f}_R F_L \quad (3.2)$$

where F is the left handed doublet containing the general fermion f and where there may in general be such an interaction for each fermion in the model. We might expect the third family fermions and their respective techni-fermion counter parts to share quantum numbers and hence horizontal interactions. Our models will respect this constraint except when direct top condensation is investigated. Again the reader may envision that these interactions are generated at the scale M_{ETC} perhaps most naively by the breaking of an additional $U(1)$ gauge group (allowing for the different fermions within a family to have different horizontal charges). We also note that all the four Fermi operators will have charges below their critical

couplings hence we may skip any discussion of the strong properties of isolated U(1) gauge interactions.

A realistic ETC scenario must agree with experimental data for m_{ν_τ} , m_τ , m_t , m_b , V,T and S. The general model has 8 independent four Fermi charges (12 if we allow the third family horizontal interactions to differ from their techi counter parts) and, therefore, we might expect solutions to exist compatible with the data. In Section 5 we will verify that such a solution exists, however, it is clearly unproductive. It is interesting to propose the minimal model in principle capable of reproducing the fermion mass spectra and test it for mass predictions.

To simplify the initial analysis of this paper we shall neglect the discussion of the neutrino masses in the model since their masses do not fit any obvious pattern in relation to the other light fermion masses. We effectively assume that there are no right handed neutrinos though we maintain right handed techni-neutrinos. The precise mechanism for suppressing neutrino masses is extremely model dependent. In addition since there is no reliable method of estimating the contribution to the S parameter from strongly interacting doublets we shall simply set the techni-neutrino mass to 50-100GeV in the future discussion and assume that a realistic S parameter is obtained provided the techni-electron mass lies between 150-250GeV as discussed in Section 2. The W and Z gauge boson masses will be dominated by the heavier techni-quarks and hence neglecting the details of the neutrino sector will have little effect on the technicolour dynamics. The T parameter, however, will presumably be dominated by the techni-lepton sector as in the techni-fermion spectra discussed in Section 2. We assume that the techni-lepton sector contributes $T \leq 1$ and hence the T parameter contribution from the techni-quarks must at most be a few tenths.

In addition we note that the CKM matrix elements only significantly vary from the identity for the first (lightest) family of fermions whose masses are generated by the weakest ETC operators. We conclude that quark mixings and CP violation are generated by those

weak interactions and, therefore, in discussion of the heavier two generations of fermions we may neglect the CKM matrix elements. There is no clear understanding of the origin of the CKM matrix elements and hence we wish to neglect their generation in this discussion since we wish to make model independent predictions. Making this approximation will clearly upset any predictions of the first family masses which are associated with large mixings and indeed in Section 6 we shall see this manifest.

Now we may consider the minimal number of ETC interactions necessary to generate the light fermion masses [29]

$$\begin{aligned}
m_t &= 160 \pm 30 GeV & m_b &= 5.0 \pm 0.3 GeV & m_\tau &= 1.784 GeV \\
m_c &= 1.5 \pm 0.2 GeV & m_s &= 0.2 \pm 0.1 GeV & m_\mu &= 0.105 GeV \\
m_u &= 5 \pm 3 MeV & m_d &= 10 \pm 5 MeV & m_e &= 0.51 MeV
\end{aligned} \tag{3.3}$$

The EWSB scale is set by the technicolour dynamics corresponding to the scale Λ_{TC} at which the technicolour group becomes strongly interacting. The third family masses are suppressed relative to this scale by a factor of ~ 10 , the second family by a further factor of $\sim 10 - 100$ and the first family by yet a further factor of $\sim 10 - 100$. It is natural to associate each generation with a separate sideways interaction (introducing a single additional interaction parameter for each family). The quarks in each family are more massive than the leptons so we must break the symmetry between them by the addition of at least one extra interaction; we shall introduce a single horizontal interaction for the quarks. Finally we notice that in the heaviest two families the top type quarks are more massive than the bottom type (for the moment we ignore the up down mass inversion since it is associated with the scale at which the approximation that the CKM matrix is the identity breaks down) and hence there must be an additional interaction on these quarks to break the symmetry between them; we introduce a single additional horizontal interaction for top type quarks.

There must be a minimum of 5 new interactions in our model to break the global symme-

tries that would otherwise leave the light fermions degenerate. Indeed it is hard to imagine how any model of the light fermion masses could have fewer free parameters than this.

4 The Devil We Know - The Gap Equation

Before we can discuss the success or failure of scenarios such as those discussed in Section 3 we must have a reliable method of calculating physical quantities in strongly interacting theories. The infinite tower of Schwinger Dyson equations are untractable so it is traditional to truncate the tower after the fermion two point function and replace other propagators and vertices with the perturbative Feynman rule. We then obtain the two gap equations [15] for the fermion self energy from SU(N) gauge interactions (in Landau gauge and with a running gauge coupling) and four Fermi interactions respectively

$$\Sigma(p) = \text{---}\overline{\times}\text{---} = \frac{3C(R)}{4\pi} \int_0^{\Lambda^2} \alpha(Max(k^2, p^2)) \frac{k^2 dk^2}{Max(k^2, p^2)} \frac{\Sigma(k)}{k^2 + \Sigma^2(k)} \quad (4.1)$$

$$\Sigma(p) = \frac{\text{---}\overline{\times}\text{---}}{g^2} = \frac{g^2}{8\pi^2\Lambda^2} \int_0^{\Lambda^2} k^2 dk^2 \frac{\Sigma(k)}{k^2 + \Sigma^2(k)} \quad (4.2)$$

where C(R) is the casimir operator of the fermion's representation of the gauge group, α the running gauge coupling, g the four Fermi interaction strength, and Λ the UV cut off.

The major success of these gap equations is that they show chiral symmetry breaking behaviour [15]. In each case there is some critical coupling below which the solution to the equation is $\Sigma(k^2) = 0$ and above which $\Sigma(k^2) \neq 0$. Clearly, however, in the case of the gauge coupling the precise value of the critical coupling and the form of the solution depend upon the form of the running of the coupling both in the high momentum regime (where the

running may be calculated in perturbation theory and is known to depend on the number of interacting fermions) and in the non-perturbative regime.

In order to investigate the consistency of solutions within the gap equation approximation let us consider the minimal predictive ETC model proposed in Section 3. The gap equations for the techni-family and third family are

Techni – electron

$$\frac{\Sigma_E}{\text{---}\times\text{---}} = \frac{\text{TC}}{\Sigma_E} + \frac{m_\tau}{g_3^2}$$

Techni – down

$$\frac{\Sigma_D}{\text{---}\times\text{---}} = \frac{\text{TC}}{\Sigma_D} + \frac{\Sigma_D}{g_Q^2} + \frac{m_b}{g_3^2}$$

Techni – up

$$\frac{\Sigma_U}{\text{---}\times\text{---}} = \frac{\text{TC}}{\Sigma_U} + \frac{\Sigma_U}{g_Q^2} + \frac{\Sigma_U}{g_t^2} + \frac{m_t}{g_3^2}$$

Tau

$$m_\tau \text{---}\times\text{---} = D(R) \frac{\Sigma_E}{g_3^2}$$

Bottom

$$m_b \text{---}\times\text{---} = D(R) \frac{\Sigma_D}{g_3^2} + \frac{m_b}{g_Q^2}$$

Top

$$m_t \text{---}\times\text{---} = D(R) \frac{\Sigma_U}{g_3^2} + \frac{m_t}{g_Q^2} + \frac{m_t}{g_t^2}$$

(4.3)

where $D(R)$ is the dimension of the techni-fermions' representation under the technicolour group.

The scale Λ_{TC} is determined by requiring the correct Z mass which is given by the techni-pion decay constant, F_π , in Eqn(2.5-7). For simplicity we neglect the mass splitting within the lepton doublet in the calculation of F_π ; since the Z mass is dominated by the techni-quark contribution to F_π this will introduce only small errors and allows us to avoid the complication of specifying the neutrino sector. The three four Fermi couplings are determined, for a given value of M_{ETC} , by requiring that the correct tau, top and bottom masses are obtained as solutions. We tune to two significant figures in the fermion masses and use $m_t \sim 170 GeV$ as a representative value. We cut the integrals off at M_{ETC} .

The dependence of the solutions on the N_{TC} , M_{ETC} and the running of the coupling both in the perturbative and non-perturbative regimes may now be investigated. We begin by allowing the technicolour coupling to run according to the one loop β -function result above Λ_{TC} and cut off the running below Λ_{TC}

$$\begin{aligned} \alpha(q^2) &= 2\alpha_C & q^2 < \Lambda_{TC} \\ \alpha(q^2) &= \frac{2\alpha_C}{1+2\alpha_C\beta\ln(q/\Lambda_{TC})} & q^2 > \Lambda_{TC} \end{aligned} \tag{4.4}$$

where α_C is the critical coupling in the fixed point theory ($\alpha_C = \pi/3C(R)$). We set $\beta = 1$, a typical running value and $M_{ETC} = 10TeV$. We assume that the techni-fermions lie in the fundamental representation of the technicolour group. In Fig 1 we show results for the techni-fermion self energies as a function of momenta for $N_{TC} = 3$ and 6. In Fig 2 we display the dependence of the techni-up quark's self energy in the $SU(3)_{TC}$ scenario to changes in the β -function for $M_{ETC} = 10TeV$. If the β -function falls below 0.22 then Λ_{TC} must be reduced below 100GeV which is presumably unphysical. In Fig 3 we show the low energy structure of the techni-up quark's self energy for varying ETC scales ($M_{ETC} = 5, 10$ and $50TeV$) again with $N_{TC} = 3$ and $\beta = 1$. The couplings that satisfy all these solutions are given in Table 1.

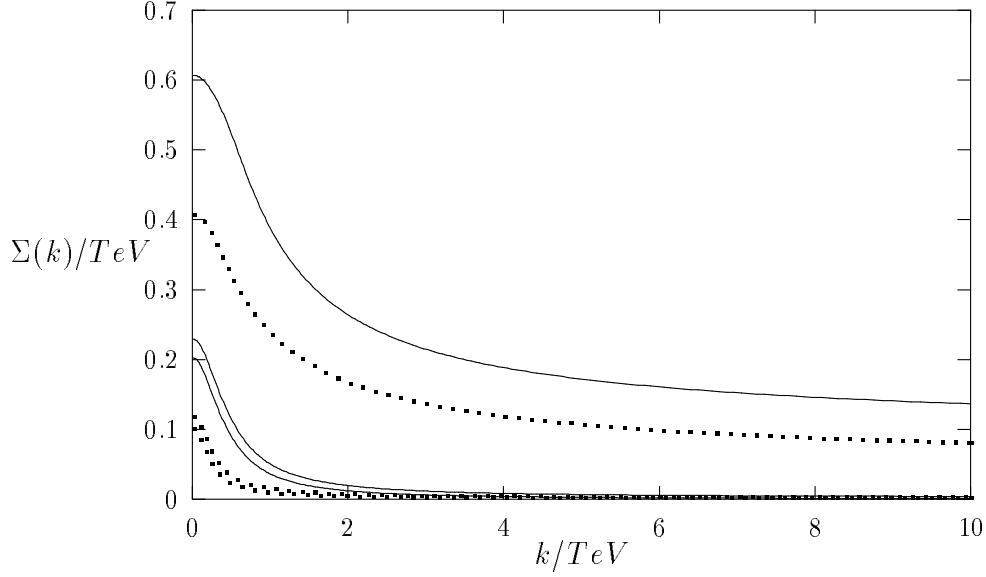


Fig 1: The solutions to the gap equations of Eqn(4.3) for the techni-fermion self energies with $N_{TC} = 3$ (solid curves) and $N_{TC} = 6$ (dashed curves). $M_{ETC} = 10TeV$ and $\beta = 1$. In each case the highest curve is the techni-up self energy, the middle curve the techni-down self energy and the lower curve the techni-electron self energy. The solutions are given by tuning the couplings to M_Z, m_t, m_b and m_τ .

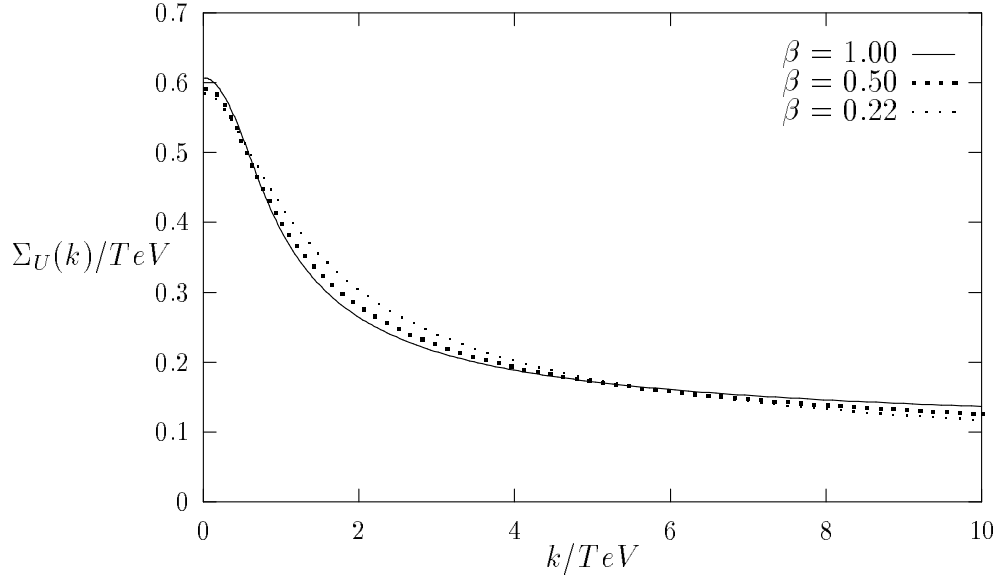


Fig 2: Dependence of gap equation solutions in Eqn(4.3) for the techni-up self energy on the technicolour β -function with $N_{TC} = 3$ and $M_{ETC} = 10TeV$.

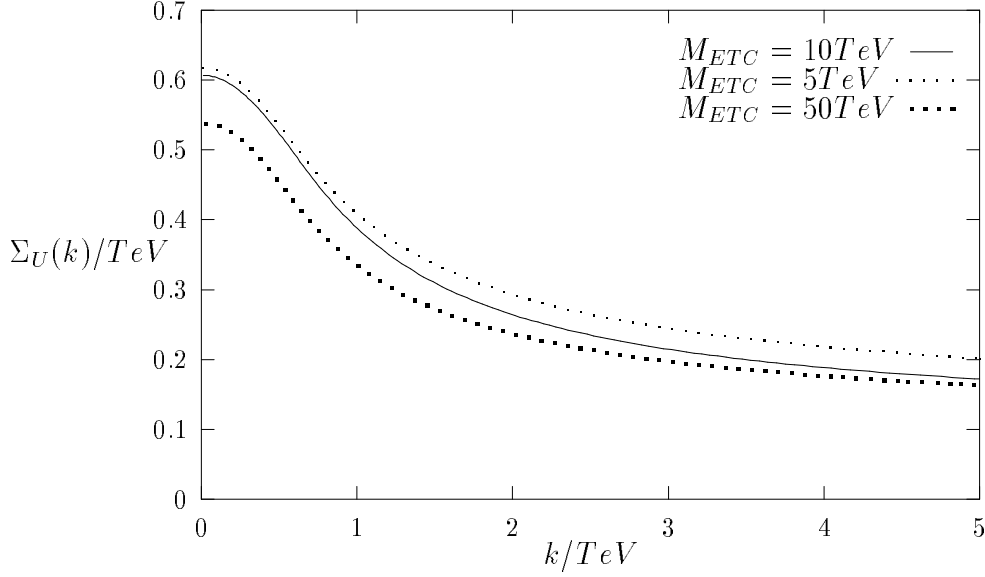


Fig 3: Dependence of gap equation solutions for the techni-up self energy in Eqn(4.3) on

M_{ETC} with $N_{TC} = 3$ and $\beta = 1$.

The ansatz for the running of α in the non-perturbative regime in Eqn(4.4) is only determined in as much as it must be finite at $q = 0$. In Fig 4 we compare the effects of two extreme choices for this regime. The first ansatz assumes that the coupling flattens out quickly at low momenta having the form of Eqn(4.4) but taking a maximum value of $1.5\alpha_C$. This ansatz is probably an underestimate of the coupling strength since there is a large discrepancy between Λ_{TC} and $\Sigma(0)$. The second ansatz assumes that outside the perturbative regime the coupling rises sharply from α_C to a maximum value of $3\alpha_C$

$$\begin{aligned} \alpha(q^2) &= 3\alpha_C & q^2 < \Lambda_{TC} \\ \alpha(q^2) &= \frac{\alpha_C}{1 + \alpha_C \beta \ln(q/\Lambda_{TC})} & q^2 > \Lambda_{TC} \end{aligned} \tag{4.5}$$

this presumably is a somewhat over estimate of the coupling strength.

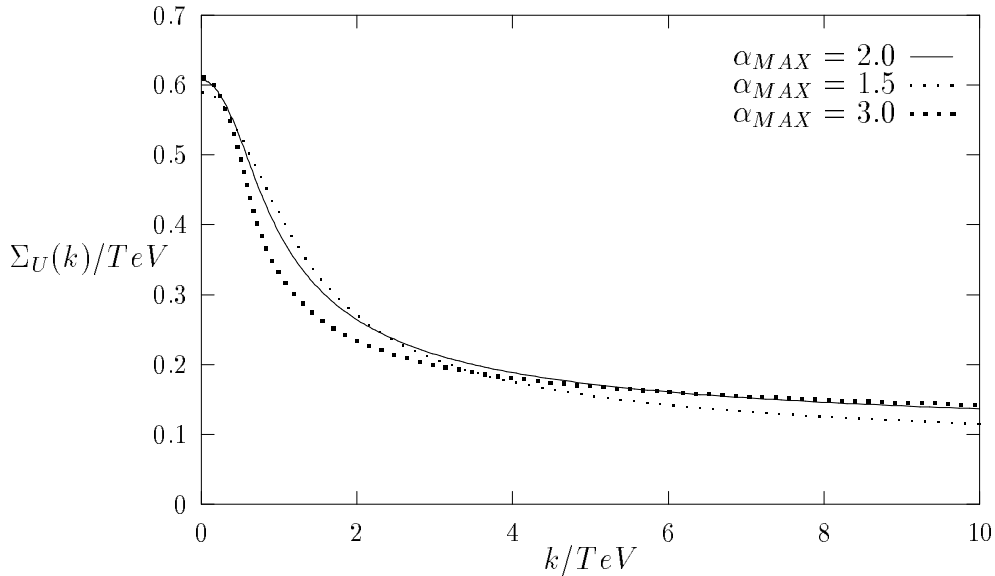


Fig 4: Dependence of gap equation solutions on the non perturbative running. Details of the coupling ansatzs are given in the text. $N_{TC} = 3$, $\beta = 1$ and $M_{ETC} = 10T eV$.

We observe that in each case the self energy solutions have the same general form though it is clearly impossible to distinguish the solutions phenomenologically. Although there is some variation in the shape of $\Sigma(k^2)$ the area under $\Sigma(k^2)$ that contributes to the light fermion masses are fixed (by the requirement that they give the correct Z mass) at least up to errors of at most order one. We therefore expect the light fermion masses we calculate in the gap equation approximation to a given ETC model to at least be representative of the rough pattern of masses the theory would produce. However, the precision electroweak parameters are plagued by error in this approximation. The T parameter is a measure of one percent differences between our calculated values of F_{π^3} and F_{π^\pm} which correspond to integrals over the self energies. Clearly this level of precision is not provided for. The calculated values for the techni-quark contribution to T in each of the above scenarios is given in Table 1 and vary between $T=8.9$ and $T=24.2$! Similarly we have argued that to achieve a realistic value of S and V we require the techni-electron mass (determined by the condition $\Sigma(M_E) = M_E$) lies in the range $150 - 250 GeV$. The calculated value of M_E is given in Table 1 also and again we see a large variation, $M_E = 90 - 260 GeV$. We shall only

be able to argue about the gross features of the techni-fermion spectra and on where these are compatible with the realistic mass spectra in Eqn(2.10).

	N_{TC}	α_{MAX}	β	M_{ETC}	Λ_{TC}	$g_3/g_C\%$	$g_Q/g_C\%$	$g_t/g_C\%$	T_Q	M_E/GeV
	3	2	1.00	10	0.60	40.0	51.8	52.66	16.4	170
FIG 1	6	2	1.00	10	0.20	13.4	33.2	35.1	20.4	90
FIG 2	3	2	0.75	10	0.50	41.6	49.5	50.1	17.5	160
	3	2	0.50	10	0.35	45.0	44.5	46.1	19.5	140
	3	2	0.22	10	0.10	52.9	29.3	36.0	24.2	90
FIG 3	3	2	1.00	5	0.52	30.4	50.64	60.8	19.0	150
	3	2	1.00	50	0.49	70.7	17.8	14.5	15.8	140
FIG 4	3	3	1.00	10	0.60	36.0	55.1	57.7	8.9	260
	3	1.5	1.00	10	1.10	48.4	38.3	47.8	22.9	100

Table 1: Numerical values of the couplings and scales used to plot Fig 1-4. The non-perturbative ansatz for the technicolour coupling is indicated by the maximum value α_{MAX} . The four Fermi couplings are given as percentages of the critical coupling ($g_C^2 = 8\pi^2$). T_Q is the contribution to T from the techni-quarks. Solutions are obtained by tuning parameters to give the correct Z mass, m_τ , m_b and m_t .

Finally we note that even if the gap equations are not a realistic approximation to the underlying Schwinger Dyson equations they still provide a parameterization of the techni-fermions' self energies. Thus whilst the gap equation couplings may not be physical the

existence of gap equation solutions consistent with the experimental data is indicative that couplings exist in the full theory also compatible with the data.

5 Successful Scenarios

Our analysis in Section 4 of the minimal predictive model proposed in Section 3 suggests that the techni-quarks in such a scenario give rise to too large a contribution to the T parameter ($T_Q \sim 15$, see Table 1). It is interesting to note however that the techni-fermion self energies (in Fig 1) show the general pattern of the realistic mass pattern in Eqn(2.10) except for the overly large splitting between the techni-up and techni-down quarks. In this section we present two scenarios in which the techni-up techni-down mass splitting lies within experimental constraints, one model is completely unproductive the other is a variation on the minimal predictive model with direct top condensation.

5.1 An Existence Proof

In principle the ETC couplings in the generalized ETC model described in Section 3 need not be related and we obtain the gap equations

Techni – electron

$$\begin{array}{c} \Sigma_E \\ \text{---} \times \text{---} \end{array} = \begin{array}{c} \text{TC} \\ \text{---} \times \text{---} \\ \Sigma_E \end{array} + \begin{array}{c} \Sigma_E \\ \text{---} \times \text{---} \\ g_E^2 \end{array} + \begin{array}{c} m_\tau \\ \text{---} \times \text{---} \\ g_\tau^2 \end{array}$$

Techni – down

$$\begin{array}{c} \Sigma_D \\ \text{---} \times \text{---} \end{array} = \begin{array}{c} \text{TC} \\ \text{---} \times \text{---} \\ \Sigma_D \end{array} + \begin{array}{c} \Sigma_D \\ \text{---} \times \text{---} \\ g_D^2 \end{array} + \begin{array}{c} m_b \\ \text{---} \times \text{---} \\ g_b^2 \end{array}$$

Techni – up

$$\begin{array}{c} \Sigma_U \\ \text{---} \times \text{---} \end{array} = \begin{array}{c} \text{TC} \\ \text{---} \times \text{---} \\ \Sigma_U \end{array} + \begin{array}{c} \Sigma_U \\ \text{---} \times \text{---} \\ g_U^2 \end{array} + \begin{array}{c} m_t \\ \text{---} \times \text{---} \\ g_t^2 \end{array}$$

(5.1)

Tau

$$\begin{array}{c} m_\tau \\ \text{---} \times \text{---} \end{array} = D(R) \begin{array}{c} \Sigma_E \\ \text{---} \times \text{---} \\ g_\tau^2 \end{array} + \begin{array}{c} m_\tau \\ \text{---} \times \text{---} \\ g_E^2 \end{array}$$

Bottom

$$\begin{array}{c} m_b \\ \text{---} \times \text{---} \end{array} = D(R) \begin{array}{c} \Sigma_D \\ \text{---} \times \text{---} \\ g_b^2 \end{array} + \begin{array}{c} m_b \\ \text{---} \times \text{---} \\ g_D^2 \end{array}$$

Top

$$\begin{array}{c} m_t \\ \text{---} \times \text{---} \end{array} = D(R) \begin{array}{c} \Sigma_U \\ \text{---} \times \text{---} \\ g_t^2 \end{array} + \begin{array}{c} m_t \\ \text{---} \times \text{---} \\ g_U^2 \end{array}$$

The top and bottom quark masses within this general model are determined by their separate sideways interactions. Although the top and bottom masses feed back into the techni-fermions' self energies tending to enhance the techni-up self energy it is clear that the separate horizontal interactions on the top and bottom type quarks can be used to enhance the techni-bottom self energy to oppose this custodial SU(2) violating effect. We can tune a set of couplings to give $T_Q = 0$ and which correctly describe the Z, tau, top and bottom

masses eg a scenario with $g_E = g_U = 0$:

N_{TC}	α_{MAX}	β	M_{ETC}	Λ_{TC}	$g_\tau/g_C\%$	$g_b/g_C\%$	$g_t/g_C\%$	$g_D/g_C\%$	T_Q
3	2.0	1.00	10	0.5	48.4	5.9	70.1	85.5	0.0

which give the techni-fermion masses

$$M_U \sim 400 GeV, \quad M_D \sim 400 GeV, \quad M_E \sim 140 GeV \quad (5.2)$$

Such a scenario is consistent with the techni-fermion mass spectrum in Eqn(2.10) and hence with all available experimental data. The renormalizable models of Ref[4] can give rise to precisely this spectrum of ETC interactions, however, the degeneracy of the techni-quarks (and hence the low T parameter) arises from a conspiracy in the four Fermi couplings which seems unnatural. Nevertheless this scenario does provide an existence proof for ETC models.

5.2 Direct Top Condensation

The minimal predictive model of Eqn(4.3) fails because the techni-up self energy must be enhanced by too much relative to the techni-down in order to generate the top bottom mass splitting. Recently there has been much discussion in the literature of direct top condensation [2] giving rise to the large top mass. Whilst top condensation on its own is plagued by difficulties of fine tuning in order not to generate too large a top mass (ruled out by the T parameter measurements) when the top is not the major source of EWS breaking such fine tuning problems need not exist. We can construct an ETC model with top condensation simply by removing the horizontal interaction on the techni-up quark in the minimal predictive model. Since the large top mass is no longer generated by the sideways ETC interactions there is less constraint upon the ETC breaking scale, M_{ETC} , from the

$Z \rightarrow b\bar{b}$ vertex measurements. We shall allow M_{ETC} to fall to 5 TeV. The gap equations are then

Techni – electron

$$\frac{\Sigma_E}{\text{---}\times\text{---}} = \frac{\text{TC}}{\Sigma_E} + \frac{m_\tau}{g_3^2}$$

Techni – down

$$\frac{\Sigma_D}{\text{---}\times\text{---}} = \frac{\text{TC}}{\Sigma_D} + \frac{\Sigma_D}{g_Q^2} + \frac{m_b}{g_3^2}$$

Techni – up

$$\frac{\Sigma_U}{\text{---}\times\text{---}} = \frac{\text{TC}}{\Sigma_U} + \frac{\Sigma_U}{g_Q^2} + \frac{m_t}{g_3^2}$$

(5.3)

Tau

$$m_\tau \text{---}\times\text{---} = D(R) \frac{\Sigma_E}{g_3^2}$$

Bottom

$$m_b \text{---}\times\text{---} = D(R) \frac{\Sigma_D}{g_3^2} + \frac{m_b}{g_Q^2}$$

Top

$$m_t \text{---}\times\text{---} = D(R) \frac{\Sigma_U}{g_3^2} + \frac{m_b}{g_Q^2} + \frac{m_t}{g_Q^2}$$

In Table 2 we show some solutions to these equations and their predictions for the contribution to the T parameter from the techni-quarks.

N_{TC}	α_{MAX}	β	M_{ETC}	Λ_{TC}	$g_3/g_C\%$	$g_Q/g_C\%$	$g_t/g_C\%$	T_Q	M_E/GeV
3	2	1.00	10	1.10	20.3	51.2	83.8	0.71	320
3	1.5	1.00	10	2.65	21.4	39.4	89.1	2.92	225
3	3	0.95	10	0.95	20.3	58.0	79.9	0.20	410
3	2	0.50	10	0.78	19.7	47.3	86.1	1.00	300
6	2	1.00	10	0.45	24.2	48.4	82.4	5.08	185
3	2	1.00	5	1.07	13.5	47.3	87.1	0.09	300
4	2	1.00	5	0.85	12.4	47.3	87.2	0.11	270
5	2	1.00	5	0.65	13.5	46.1	87.8	0.24	230

Table 2: Numerical values of the couplings and scales of solutions to Eqn(5.3). The non-perturbative ansatz for the technicolour coupling is indicated by the maximum value α_{MAX} . The four Fermi couplings are given as percentages of the critical coupling ($g_C^2 = 8\pi^2$). T_Q is the contribution to T from the techni-quarks. Solutions are obtained by tuning parameters to give the correct Z mass, m_τ , m_b and m_t .

The solutions with a low ETC scale seem consistent with the techni-fermion mass spectrum proposed in Section 2 though the techni-electron mass is somewhat high. Within the gap equation approximation it is certainly not possible to discount this scenario so we shall consider it a successful ETC model.

6 Quark Mass Postdictions

We have argued in Section 3 that a model of EWSB and the third family masses (excluding neutrinos) must have at least four couplings and hence can not be “postdictive” of the third family masses. However, it is conceivable that only one additional parameter need be added to generate the second family masses (a parameter that suppresses the second family masses relative to the third) since quark lepton and custodial isospin symmetry breaking already exist in the model. Similarly one additional parameter might suffice to suppress the first family masses below the second but of course our neglect of the CKM matrix elements which are substantial for the first family makes this seem less likely to be successful. In this section we investigate the possibility of such postdiction in the scenarios we have discussed above.

The “existence proof” scenario does not lend itself to postdiction since to follow the pattern of the model of the third family masses we could simply introduce additional sideways interactions for each new light fermion sufficient to generate their mass. There are no constraints on the couplings so they are unpredictable. The first and second family masses are at least two orders of magnitude smaller than the techni-fermion masses and hence any feedback of the light two families masses into the techni-fermion self energies are negligible and do not upset our calculations of S and T. Although unpredictable the scenario still provides an existence proof of a realistic ETC model.

The top condensation scenario however is potentially predictive as described above. We introduce the additional sideways interactions

$$\begin{array}{c} \text{Muon(Electron)} \\ m_{\mu(e)} \end{array} \quad \begin{array}{c} \text{---} \times \text{---} \end{array} = D(R) \quad \begin{array}{c} \Sigma_E \\ \text{---} \bigcirc \text{---} \\ g_{2(1)}^2 \end{array} + \begin{array}{c} m_\tau \\ \text{---} \bigcirc \text{---} \\ g_{2(1)}^2 \end{array}$$

Strange(Down)

$$\begin{array}{c} m_{s(d)} \\ \text{---} \times \text{---} \end{array} = D(R) \left(\begin{array}{c} \Sigma_D \\ \text{---} \text{---} \text{---} \\ g_{2(1)}^2 \end{array} + \begin{array}{c} m_b \\ \text{---} \text{---} \text{---} \\ g_{2(1)}^2 \end{array} + \begin{array}{c} m_{s(d)} \\ \text{---} \text{---} \text{---} \\ g_Q^2 \end{array} \right) \quad (6.1)$$

Charm(Up)

$$\begin{array}{c} m_{c(u)} \\ \text{---} \times \text{---} \end{array} = D(R) \left(\begin{array}{c} \Sigma_U \\ \text{---} \text{---} \text{---} \\ g_{2(1)}^2 \end{array} + \begin{array}{c} m_t \\ \text{---} \text{---} \text{---} \\ g_{2(1)}^2 \end{array} + \begin{array}{c} m_{c(u)} \\ \text{---} \text{---} \text{---} \\ g_Q^2 \end{array} \right)$$

which we would expect to be generated if there was a single breaking scale associated with each of the first and second families in the breaking of $SU(N+3)_{ETC} \rightarrow SU(N)_{TC} +$ three families. Again the feedback of the first and second family masses to the techni-fermions and third family are negligible. We set the coupling strength of the new sideways interaction by requiring that we generate the correct muon and electron masses. The up, down, charm and strange quark masses are now predictions of the model. Explicitly

$$\begin{array}{ll} \Lambda_{TC} & \text{determined by } M_Z \\ g_t & \text{determined by } m_t \\ g_3 & \text{determined by } m_\tau \\ g_2 & \text{determined by } m_\mu \\ g_Q & \text{determined by } m_b \\ g_1 & \text{determined by } m_e \end{array} \quad (6.2)$$

Although the predictions of the model are clear cut our ability to calculate is limited as discussed in Section 4. The gap equation solutions are, however, moderately well bounded since the integrals over the techni-fermion's self energies are fixed to a good degree by the imposed requirements that they correctly give the Z, tau, bottom and top masses. We shall quote the range of predictions from all the coupling values in Table 2 as an estimate of our theoretical errors. We obtain

$$\begin{aligned}
m_c &= 1.5 \pm 0.8 GeV, & m_s &= 0.32 \pm 0.02 GeV \\
m_u &= 6.6 \pm 3.7 MeV, & m_d &= 1.5 \pm 0.2 MeV
\end{aligned}
\tag{6.3}$$

We immediately notice that these predictions are in surprisingly good agreement with the observed mass spectra except for the down quark. The failure to predict the down quark mass however is to be expected since we have neglected the generation of the CKM matrix which has large elements for the first family. Conservatively we can conclude that ETC models with the minimal number of ETC interactions that are sufficient to break the global symmetry of the light fermions in the observed pattern seem capable of reproducing the pattern of the observed light fermion mass spectrum.

7 Conclusions

The precision data from LEP [8] has provided tight constraints on the form of models of EWSB. It has been argued [13, 14] that technicolour models with a single techni-family with a light techni-neutrino and degenerate techni-quarks give contributions to the S,T and V parameters that lie within the experimentally allowed bands. If the top mass is generated by strong ETC interactions broken above $10 TeV$ then the model will lie within the experimental limits on non-oblique corrections to the $Zb\bar{b}$ vertex as well [12]. As a first step towards a fully renormalizable, predictive model of EWSB we have investigated whether an ETC model can be compatible both with the precision data and the light fermion masses. To make this investigation we have used a generalized one family ETC model in which the ETC interactions are represented by four Fermi interactions.

To calculate within this generalized model we have used the gap equation approximation to the Schwinger Dyson equations. Unfortunately even within the gap equation approximation the solutions for the techni-fermions self energies, $\Sigma(k^2)$, are dependent on the precise form of the running of the technicolour coupling. The technicolour dynamics are fixed to some degree by the requirement that the model gives rise to the correct Z boson mass (given

by an integral equation over the self-energies). Calculation of the light fermion masses (also given by integral equations over the self-energies) are, therefore, moderately stable. However, the precision electroweak variables are very sensitive to shifts in for example $\Sigma(0)$ and are hence less well determined. Nevertheless we have argued that couplings exist in the generalized ETC model that very plausibly fit the experimental constraints.

Two scenarios in the generalized ETC model have been found consistent with the precision data and the third family fermion masses. The first is an unproductive existence proof in which sufficient ETC couplings are included that the fermion mass spectra may be tuned to match the data. The second scenario contains what we have argued is the minimum number of different strength ETC interactions required to break the global symmetry on the third family in the observed pattern. This model achieves a sufficiently large top mass by direct top condensation.

In order to obtain a large top mass in these models the ETC interactions must be tuned close to their critical values. The “fine tuning” is at worst of order 10%, corresponding in our results to our need to quote ETC couplings to three significant figures in order to tune to two significant figures in the light fermion masses. In fact the tuning is only this severe for the ETC couplings that generate the top mass. This degree of tuning may not be unnatural since gauge couplings naturally run between their critical value, g_C , and $\sim 0.1g_C$ over many orders of magnitude of momentum. Clearly any greater degree of fine tuning which, for example, would be associated with significantly increasing Λ_{ETC} , would be unsatisfactory.

The top condensing scenario may be minimally extended to the first and second families. The model then makes predictions for the up, down, charm and strange quark masses. Our calculation of these masses shows that the charm, strange and up quark mass predictions are consistent (up to errors due to uncertainty in the gap equation approximation) with the experimental values. The model does not reproduce the up down mass inversion observed in nature but we have argued that this is the result of our neglect of the mechanism for

the generation of the CKM matrix which has large elements for the first family quarks. In addition we have neglected a discussion of the neutrino sector since their masses do not fit any obvious pattern in the fermion mass spectra. In this paper we have concentrated on predictions which are potentially generic to ETC models. Clearly it would be of interest to continue the analysis to models of neutrino masses and the CKM matrix but such analysis would only serve to confuse the cleaner model of quarks and charged leptons.

Hopefully the successes of the generalized ETC model here will be translatable to a renormalizable ETC model. In this respect the proposal in Ref[15] that the quark lepton mass splittings may result from QCD interactions, corresponding to $g_Q^2 \rightarrow \alpha_{QCD}$ in the top condensate scenario, is appealing. At the EWSB scale $\alpha_{QCD}(M_Z^2)/\alpha_{QCD}^{crit} \sim 15\%$. Our analysis suggests (see Table 2) that g_Q/g_C needs to be of order 50% however. The value of g_Q/g_C can be reduced (see Table 1) by increasing the maximum value the technicolour coupling reaches in the non-perturbative regime, or by increasing N_{TC} or Λ_{ETC} or finally by decreasing the technicolour β -function towards a walking value. Unfortunately each of these changes tends to increase the T parameter contribution from the techni-quarks. The uncertainties in the gap equation analysis though does not preclude the possibility.

We conclude that our unpredicitive model provides an existence proof that ETC models exist which satisfy the stringent precision measurement bounds. The scenario with direct top condensation provides the tantalizing possibility that ETC models can be constructed that are predicitive.

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